Software System Design and Implementation

Machine-checked Properties

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Checking properties

- Logical properties are key to specifying the intended meaning of programs
    - Types,
    - QuickCheck properties,
    - Hoare triples,
    - and so on
- How can computers help to check these properties?

Checking properties dynamically

- Property-based testing (QuickCheck)
  - Properties as program fragments
  - Randomised test case generation

Checking properties dynamically

- Assertions
  - Assertions in *design by contract* (Eiffel, D, Ada)
    - specify pre- & postconditions of methods
    - invariants of objects
    - assertions can be extracted as documentation
  - Evaluating properties during program execution (testing & debugging)

Checking properties statically

- Proof checkers (theorem provers)
  - In general, (at least parts of) proofs need to be supplied manually
- Static analysis
  - Abstract interpretation, flow analysis, and so on
- Type checking
  - Types are properties
  - Type checking is a form of theorem proving (decidable logic)

Hybrid approaches

- Contracts
  - May be checked statically or dynamically
  - Possibly static checking delaying checking of residual properties until runtime
- Gradual typing
  - Statically checks for type errors in some parts of a program
  - Leaves other parts to be checked dynamically

Compiler integration provides extra leverage

- Compiler-checked properties are automatically checked on every compiler run
    - Cannot diverge from source code
    - Provide checked documentation
- Types
    - Are used and understood by every develope
    - Are tightly integrated with the language

Let's look at some particularily expressive types

Generalised Algebraic Data Types

GADTs

- Also called indexed data types
- Use of a type argument to specify a property of the data type
    - E.g., a datatype of expression terms with the type of the expression as a type argument
- Simultaneously restricts the values of the GADT
    - E.g., a list type indexed by the length of the list

Motivating example: a type-safe evaluator

```
data Expr
  = BConst Bool
  | IConst Int
  | Times Expr Expr   -- arguments must be of type Int
  | If    Expr Expr Expr   -- 1st argument must be a Bool,
                           – 2nd & 3rd of same type
```

Informal type constraints

```
eval :: Expr -> Result
```

Expression evaluation

```
data Value = IVal  Int
           | BVal Bool
```

Evaluation result

```
data Expr
 = BConst Bool
 | IConst Int
 | Times Expr Expr
 | Less  Expr Expr
 | And   Expr Expr
 | If    Expr Expr Expr
```

```
data Result
  = IVal Int
  | BVal Bool
```

```
eval :: Expr -> Result
eval (IConst n)
  = IVal n
eval (BConst b)
  = BVal b
eval (Times ex1 ex2)
  = case (eval ex1, eval ex2) of
      (IVal n1, IVal n2) -> IVal (n1 + n2)
      (_, _)             -> error "illegal expr"
    …
```

- The evaluated expressions are dynamically typed (like, say, Python programs)
- During evaluation, we check that operators (e.g., addition) receive operands of compatible type
- If the types are not compatible, we yield a runtime error (or exception)

Strongly typed languages:

`eval`

`Expr` → `Result`

precondition:
input is an `Expr`

postcondition:
input is a `Result`

invariant:
any value of type `Expr`
indeed valid object of that type.
E.g.,
`BConst 5`
statically excluded

Type checker ensures

- precondition observed whenever function is called
- invariants hold at any time during program execution
- postcondition holds after every call of the function (no guarantee for non-termination, or in case of run-time error)

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To get the most out of the type checker, we need to

- make functions total, if feasible
- make types as precise as possible

~~`If (IConst 5) (IConst True) (IConst 2)`~~

Key idea

- Parametrise expressions by the type of value they evaluate to
  - Expressions have unique types (don't change during evaluation)
  - Expression of type `τ` evaluates to a value of type `τ`
- In Haskell: `Expr t` is an expression of type `t`

Define type expressions as a data type & adapt the evaluator

Type indices

- The type argument **`t`** in `Expr t` is a type index
  - Type indices constraint the formation of values
  - The type checker rejects malformed terms; e.g.,

```
If (Const 1) (Const 2) (Const 3)   -- type error!
```

  - Our expressions can only have the types `Expr Int` and `Expr Bool`

```
If    ::          Expr Bool -> Expr s    -> Expr s -> Expr s
```

Calculating with types

Singleton types

- Indexed type, where the type index uniquely identifies the value
    - As types are sets of values, singleton types are one-element sets
- Let's look at an example: singleton Booleans

Vanilla Booleans

```
data Bool where
  False :: Bool
  True  :: Bool
```

Singleton Booleans

```
data SBool (b :: Bool) where
  SFalse :: SBool False
  STrue  :: SBool True
```

- If a function returns a value of type `SBool False`, we know the return value without executing the function (modulo non-termination)

- Singleton types enable us to reflect values to the type level

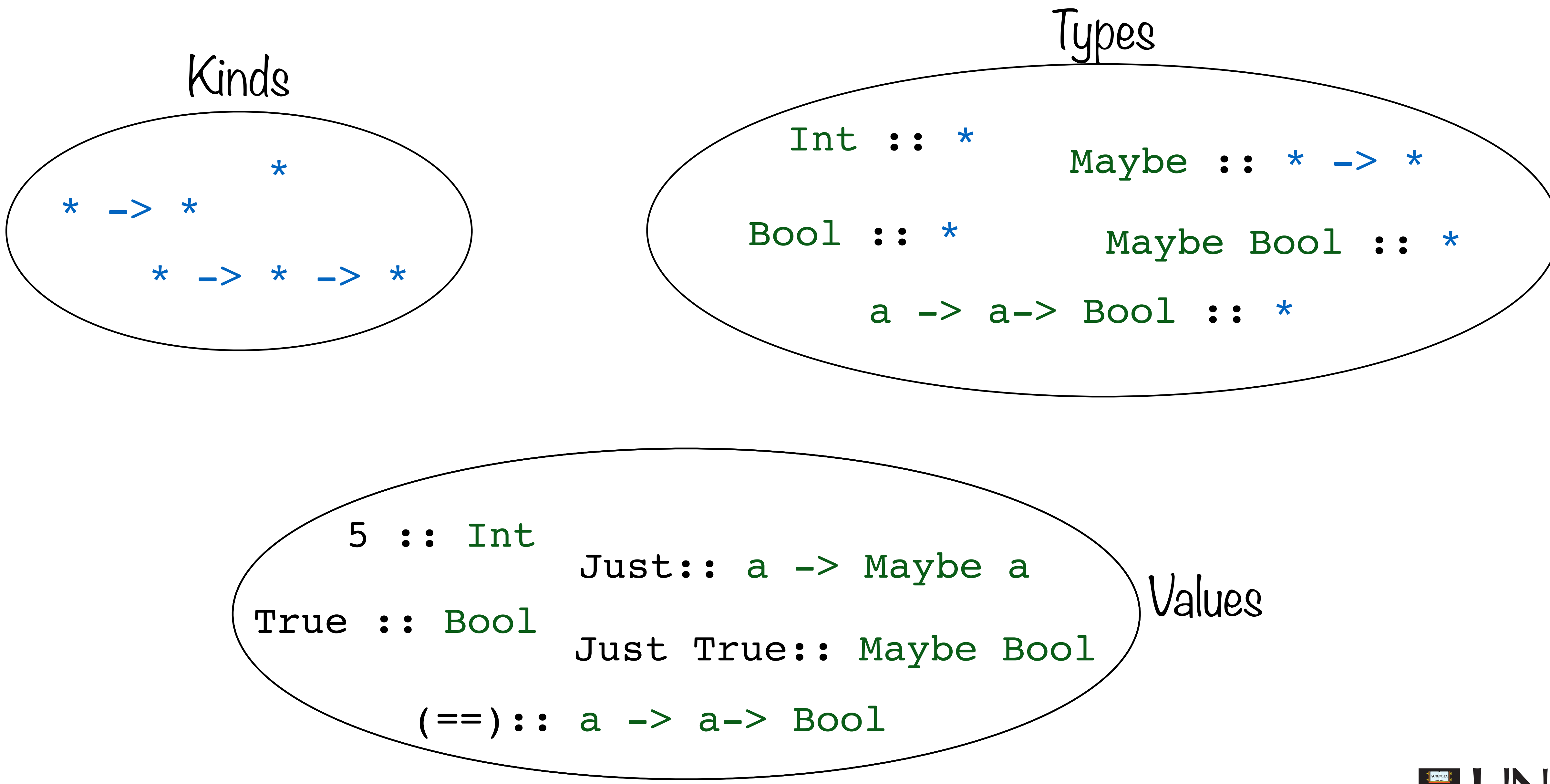
- Why is this useful?

  - Stronger types characterise the behaviour of a program more precisely

  - Haskell type checker as proof checker

Values, Types, and Kinds in plain Haskell

- Values (including functions and data constructors) have types
- Types and type constructors have kinds



Singleton natural numbers

- We can characterise the set of natural numbers inductively as follows
  - Zero (0) is a natural number
  - If *n* is a natural number, the successor of *n* (*n* + 1) is a natural number
- This characterisation is based on the Peano axioms of natural numbers

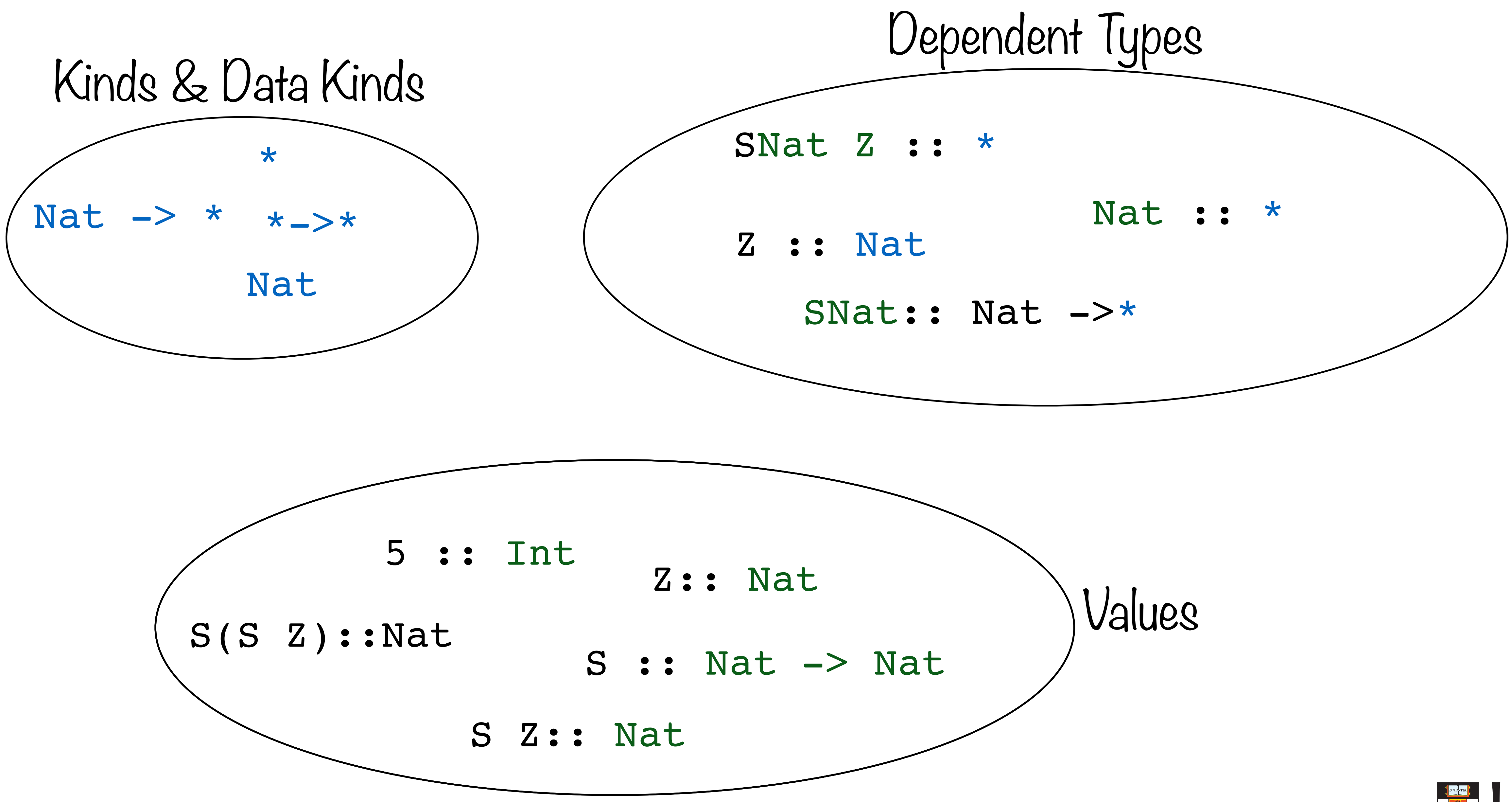
```
data Nat where
  Z :: Nat
  S :: Nat -> Nat
```

```
data Nat
  = Z
  | S Nat
```

```
data SNat (n :: Nat) where
  Zero :: SNat Z
  Succ :: SNat m -> SNat (S m)
```

With data kinds:

- Now, kinds and types overlap (e.g., Nat can be used both as a type and a kind)



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Let's take a step back — types versus values

- Types are static; values are dynamic
- Type erasure property: types don't impact a program's semantics
- Types characterise part of a programs behaviour:
    - Each value has a unique type, but usually a type stands for many values
    - In contrast: a singleton types has a unique value
- Singleton types lift data from the value to the type level
- How about computations (functions) on the type level?

Type families

- There are two forms of type families in Haskell
    - Type synonym families: effectively provide functions on types
    - Data type families: essentially are a form of open (or, extensible) GADTs
- We will focus on type synonym families, which differ from value functions:
    - They need to be terminating — how do we know (halting problem)?
    - Limited syntax and obviously no side effects
    - They are extensible (like type classes)

Computing with types

- With type families, we can define arithmetic operations on type-level numerals

- We can also tie type-level to value-level computations

Addition on SNat

```
type family (+) (n :: Nat) (m :: Nat) :: Nat
```